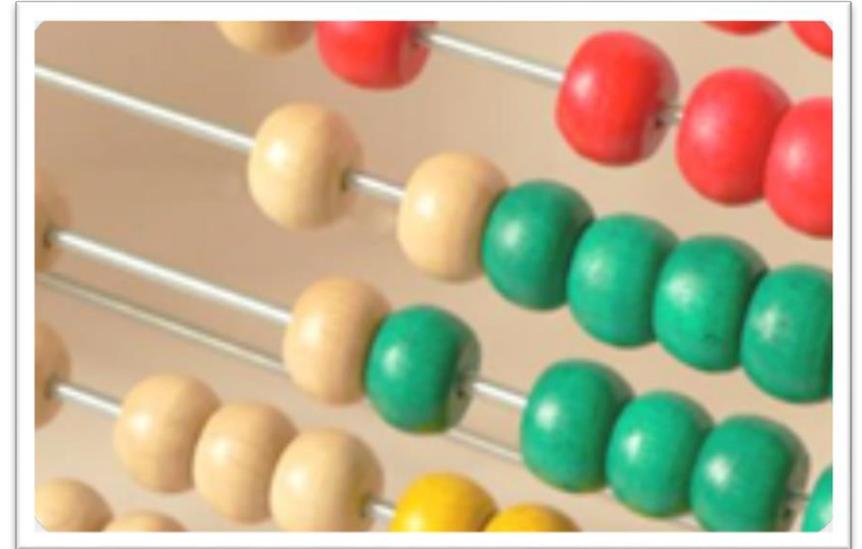


PRIMARY ADVANTAGE

Maths Programme

A Model of Best Practice

Third Edition



SUPPORTED BY

MAYOR OF LONDON



Department
for Education

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Introduction



At the individual level, mathematics underpins many aspects of our everyday activities, from making sense of information in the newspaper to making decisions about personal finances.

A good understanding of basic mathematics is essential wherever calculations, measurements, graphical interpretations and statistical analyses are necessary.

At Primary Advantage schools a substantial amount of time is devoted to the teaching and learning of mathematics to build a strong foundation for the acquisition of mathematics knowledge and skills in later years.

Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject. Achieving mastery means acquiring a solid enough understanding of the maths that's been taught to enable pupils to move on to more advanced material. (NCETM 2016)

At Primary Advantage, we want our pupils to truly master mathematical skills, to have the strongest understanding and experience applying the skills they have learnt across a wide range of contexts.

We want to ensure the learning has “stuck”. Through systematically teaching the objectives within the programme, alongside our assessment model of Prove Its (low stakes assessment activities designed to ensure that previously mastered learning is regularly revisited) we ensure that pupils have the opportunities they need to connect the mathematical ideas together.

Effective learning of mathematics requires a coherent and well-structured syllabus, excellent instructional materials, and excellent teachers who use sound pedagogical strategies that are developmentally appropriate. This is at the core of the Primary Advantage Programme. Our mathematics curriculum emphasises conceptual understanding, skills proficiency, learning of process skills and focuses on mathematical problem solving.

In sharing our experience in the development of this maths curriculum we hope to have made a positive contribution to Primary Mathematics teaching.

Acknowledgements: The programme was developed by a group of teachers from Primary Advantage Schools and without their expertise and professionalism the syllabus could not have been completed. We would like to thank Gemma Meharg, Catherine Thomas, Stephanie Saviddes, Sarah Jameson, Joanne Smith, Jo Stonehouse, Aidan Stallwood and Alyson Tyler for their input and positivity throughout the project.

We would like to thank Lucy Blewett for the 2015 edition of the programme with further thanks to Toni Mason, Anna Case, Matthew Stevenson and Izabela Jelonek for their support and input.



Aims and distinctive features

This programme aims to support you in developing the three key areas of mathematical subject knowledge.

These are:

- **Mathematical knowledge**

The programme and CPD will support in developing understanding of mathematics

- **Curriculum knowledge**

By following the programme you will come to learn exactly which areas of mathematics should be taught to each group of children you may be working with

- **Pedagogical knowledge**

The programme draws on models and images so that you can see the best ways to introduce learners to particular mathematical ideas.

The programme is practical and models best practice in primary mathematics teaching.

It will support you in the planning and delivery of lessons for the full primary range.

Distinctive features

Progression

The learning progressions within each strand are built on each other. This will help you understand which mathematical ideas should be taught to which age group of children and how the Mathematics Curriculum is developed over the primary age range.

Key concepts

These sections deal with the ideas which underpin each particular strand of mathematics covered. This allows you to see the big picture immediately and understand how the different strands knit together.

Models and images

These sections deal with the best models and images to represent the elements of mathematics in each strand. This will help you in choosing appropriate representations when planning mathematics lessons. Examples of the bar model will help you when planning for problem solving in your class.

The overview of the PA Maths Programme

Programme aim

The PA maths Programme aims to empower young people to achieve their potential, to use their knowledge of mathematical language to talk about their work and explain their findings, and ultimately use the skills they have mastered to make informed and responsible choices throughout their lives.

Infusing							
Curriculum Objectives	Fluency in the fundamentals of mathematics		Reason mathematically			Solve problems	
Cross Curricular Skills	COMMUNICATION		USING MATHEMATICS			ICT	
Thinking Skills and Personal Capabilities	Managing information, working with others		Thinking, problem solving, Decision Making, Self-management			Being creative	
Incorporating							
Assessment for Learning	Cleared learning intentions shared with pupil	Shared/negotiated success criteria	Ownership of learning	Taking risks for learning	Peer and self-assessment / evaluation of learning	Celebrating success	Advice what to improve and how to achieve
Promoting/Encouraging							
Learning Experiences	Investigating and problem solving	Curriculum links	Relevant and enjoyable	Media—rich	Skills integrated	Active and hands on	Offers choice
	Challenging and engaging	Supportive environment	Culturally diverse	Positive reinforcement	Varied	On—going reflection	Enquiry based
Fostering							
Attitudes and Dispositions	Personal responsibility	Concern for others		Commitment	Determination	Open to new ideas	Respect
	Self-confidence	Curiosity	Collaboration	Flexibility	Resourcefulness	Resilience	

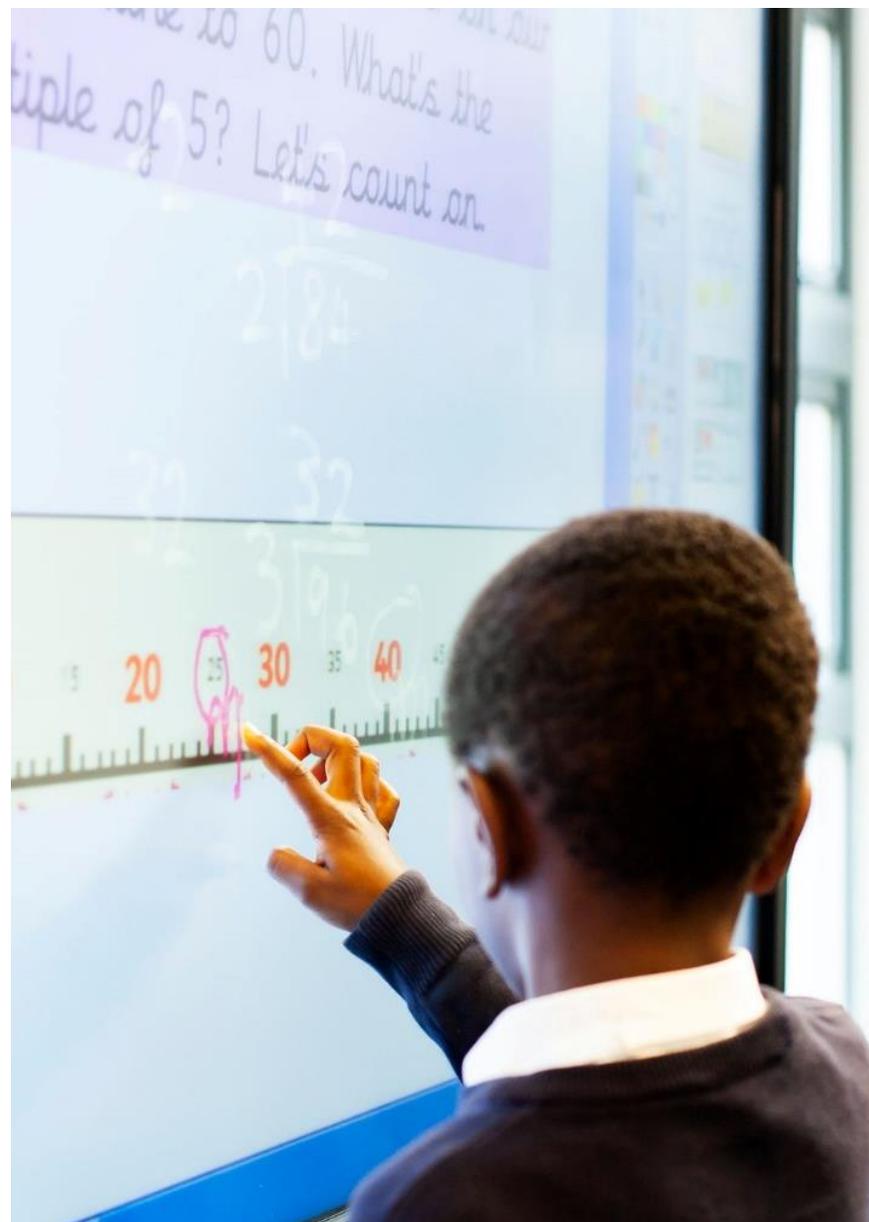
Primary Advantage Learning Dispositions

Learning mathematics extends beyond learning concepts, procedures, and their applications.

It also includes developing a disposition toward mathematics and seeing mathematics as a powerful way for looking at situations.

Disposition refers not simply to attitudes but to a tendency to think and to act in positive ways. Students' mathematical dispositions are manifested in the way they approach tasks – whether with confidence, willingness to explore alternatives, perseverance, and interest – and in their tendency to reflect on their own thinking. The assessment of mathematical knowledge includes evaluations of these indicators and students' appreciation of the role and value of mathematics.

As part of the Primary Advantage programme we encourage and reward the following standards in all of our lessons.



Primary Advantage CPD

This innovative programme is designed to support teachers in developing children's mathematical understanding and enjoyment throughout the primary phase. It has been designed by a range of classroom teachers and is a model of best practice, based on experience and theoretical understanding.

The PA Maths Programme comprises of a range of complementary elements which all contribute towards the aim of high quality primary maths teaching and learning:

- 4 core training CPD sessions
- Induction training for staff new to using the programme
- 'How to' sessions, TA subject knowledge sessions and EYFS training sessions
- The opportunity to attend modelled lessons within the PA schools
- Access to the online Maths Portal (including this supporting document)

Through engaging with the programme, teachers' planning and lesson delivery will develop in the following ways:

Mathematical knowledge	A deeper understanding of the underpinning structures of primary mathematics.
Curriculum knowledge	A secure understanding of best practice in terms of progression and challenge through curriculum planning.
Pedagogical knowledge	A deeper understanding of important models and images to support children's progress through a range of key mathematical concepts.



By engaging with the PA Maths Programme, teachers are demonstrating a commitment to sharing the values that underpin it, namely the belief that primary mathematics is a crucial phase in laying the foundations for lifelong numeracy skills. This drives the focus on conceptual understanding, skills proficiency, problem solving, reasoning and fluency which comprise the PA Maths Programme.

Through sharing our experiences with the wider primary community through the PA Maths Programme, we hope to make a positive contribution towards maths teaching and learning and thank you for joining us on our journey.

Problem Solving, Reasoning and Fluency

As its underpinning aims, problem solving, reasoning and fluency are at the heart of the National Curriculum for England and Wales (DfE, 2014). By highlighting them in this way, the DfE have indicated that they should underpin the curriculum by threading through all of the teaching and learning. By using them as a lens through which to teach the content, the national curriculum will be taught in its intended manner.

The PA Maths Programme supports this approach and believes that through developing children's problem solving, reasoning and fluency skills, there will be a range of positive outcomes, including the development of children's conceptual understanding, their ability to use maths in meaningful ways and positive attitudes from the EYFS to Year 6.

Problem Solving

This can be summarised as the ability to apply mathematics to a variety of situations (Cockcroft, 1982) and PA encourage the use of 'low threshold, high ceiling' activities. These mathematical activities are designed so that the great majority of the group can begin, and then work on at their own level of engagement, but which has lots of possibilities for the participants to do much more challenging mathematics (McClure, ND). They can lead to the development of a community of practice, positive attitudes and progression through deepening subject knowledge, rather than accelerating. There are a wealth of activities on the NRich website (www.nrich.maths.org), alongside others, which can be used alongside this PA document.

Reasoning

Reasoning can be considered the glue which holds maths together. A focus on the mathematical process and a real commitment to children's understanding, as distinct from any final product, enables the development of reasoning skills in the primary classroom. The 'example questions' section of the pages in this document help support this aspect of teachers' planning and, when used to encourage children to move from describing to explaining to justifying, it is another tool with which to challenge the higher attaining children.

Fluency

Developing children's mathematical fluency demands a focus on their efficiency, accuracy and flexibility. It requires them to know why they are doing what they are doing, and to make appropriate choices (from a toolkit of mental calculation strategies, for example). By using manipulatives (as part of a CPA approach), encouraging children to discuss their work, particularly through reasoning, and consolidating understanding across a range of meaningful contexts, children's fluency skills will develop.

These three aims are inter-related and complementary. They inform the PA Maths Programme and are deemed to be at the root of high quality maths teaching and learning.

Cockcroft, W H (1982) *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*, London, Her Majesty's Stationery Office

CPA in the PA Maths Programme

Why?

A commitment to CPA is intrinsic to the PA Maths Programme. It informs the pedagogy and planning of teachers using this programme as PA believe it is a supportive way of developing children's deep conceptual understanding, good progression and positive attitudes to maths.

What?

CPA is an approach to teaching mathematics based on the work of Jerome Bruner (1960). Bruner's premise was that children's conceptual understanding develops from being actively engaged in their learning and making sequential process through three stages of representation: enactive, iconic and symbolic (mapped onto concrete, pictorial, abstract respectively). Each stage builds on the previous one, although unlike Piagetian theory, they are not age-related.

CPA therefore encompasses multiple models that approach a concept at different cognitive levels. Firstly at the concrete level, children are exposed to a range of appropriate manipulatives, for example, dienes, unifix, Numicon, egg boxes, counters, shapes, coins and dice. Use of these concrete objects engages children with their learning and can provide a 'hook' into the learning. Another advantage of this approach is that discussion is a natural by-product of active learning which is an element of good quality maths teaching and learning (Williams, 2008).

Progress into the pictorial phase is consequently underpinned by active, memorable experiences leading to deep learning. This second phase aids visualisation and the bar model is a key element of the pictorial phase of problem solving (this is explored later).

It is important to note that although the ultimate aim of a CPA strategy is to culminate in a fluent, abstract approach characterised by quick, efficient methods, the process should not be rushed. It may be necessary to return to previous phases to address children's misconceptions and consolidate their conceptual understanding.

Another key feature of the CPA process, is that although concrete objects may be perceived as too elementary for upper KS2 children (Sousa, 2007), both concrete and pictorial representations should be used at across the primary phase.

How?

Within the PA core CPD sessions, explanations and examples of appropriate CPA apparatus, models and images are consistently discussed. They also form the backbone of this document as each year group has clear diagrams demonstrating the progression from the concrete to pictorial to abstract. This structure informs teachers' planning and pedagogy, as does reflection on observations of any modelled lessons attended.

Two key facets of the CPA approach (the counting stick and the bar model) are discussed in depth in the following pages.



CPA Exemplified

Whilst choices around which concrete and pictorial representations are used within lessons must remain the choice of each teacher, dependent upon their individual context, there are two which PA would advocate as useful across a range of learning experiences – the counting stick and the bar model.

Counting stick

This piece of concrete apparatus embodies the ubiquitous pictorial model of the number line. Traditionally it has been used for counting on/back in ones/tens and for ordering numbers, but the counting stick is a versatile piece of equipment which can be used across a range of mathematical areas to support children's fluency and understanding.

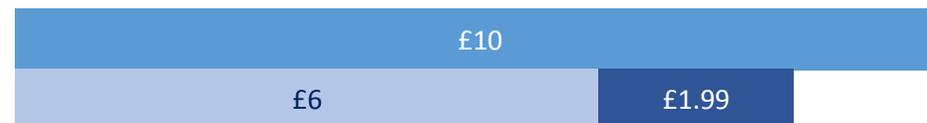
A focus on fluency is often the genesis of counting stick use, for example children counting forwards and backwards in multiples of a given number. However, by labelling the ends of the counting stick with two numbers and asking children to label another given point, their reasoning skills are required. Careful questioning can encourage them to move from giving a basic answer to explaining and then justifying their thinking. Contextualising this by using a counting stick vertically to represent scales of temperature, length or mass can be engaging and meaningful for children.

Ideas of equivalence can be developed and consolidated through using a number line too as children begin to explore other possible labels for given points such as $\frac{1}{4}$, 0.25 and 25%. The counting stick frequently features in the models and images pages throughout this document.

Bar Model

The 'bar model' is shorthand for a systematic method of representing word problems and number relationships. It is exemplified by children sketching rectangular bars to represent relationships between known and unknown numerical quantities. In this way, it can help children identify which calculation is needed to solve a word problem or investigation. It is often this step which is problematic for children, rather than the calculation itself, so the bar model, and the discussion which supports it, can be very useful.

E.g. Ali has £10 to spend on books. He chooses one for £6 and one for £1.99. How much change does he get from his £10 note?



The bar model is a visual, flexible strategy which children can fit into their 'toolbox' of heuristics for problem solving. It can be supported by concrete apparatus such as Cuisenaire rods or unifix cubes, to enable children to access and enjoy mathematical problem solving. The bar model frequently features in the models and images pages throughout this document.

Yearly Overviews



EYFS 1

EYFS 2

EYFS 1						EYFS 2						
1	2	3	4	5	6	7	8	9	10	11	12	
Shape, Space and Measures	Can say what is different and what is the same. M	Begins to categorise objects according to properties such as size (colour.) M Begins to categorise objects according to properties such as shape. GS	Begins to use the language of size. M Shows an interest in shape and space by making arrangements with objects. M	Experiments with capacity. (Which holds more/less) M Begins to talk about the shapes of everyday objects, e.g. 'round' and 'tall'. GS	Anticipates specific time-based events such as mealtimes or home time. M Understands some talk about immediate future, e.g. 'later' or 'soon'. M Understands some talk about immediate past e.g. 'before'. M Uses money in role play. M	Exchanges money for objects. M Shows awareness of similarities of shapes in the environment. Uses familiar objects and common shapes to build models. GS Beginning to use mathematical names and 'flat' 2D shapes. GS	Uses positional language (below, above, next to, beside, in front, behind and on top) GP	Describes their relative position such as 'behind' or 'next to'. Uses mathematical terms to describe 2d shapes. GS	Orders two items by mass. (using everyday language) M Uses everyday language to solve problems. M They recognise, create and describe patterns. To count patterns. Mx Orders two or three items by length or height. M	Orders two items by capacity. (using everyday language) M Uses everyday language to compare quantities & objects. M Uses everyday language to talk about distance. M Orders and sequences familiar events. M Uses everyday language related to time (begins to identify o'clock) M	Measures short periods of time in simple ways. M Uses everyday language to talk about money. M Demonstrates understanding that £1 has greater value than pennies. M	Know and name different coins – 1p, 2p, 5p, 10p, 20p, 50p, £1 & \$2. M Can use 1p, 2p, 5p & 10p coins to make amounts up to 20p. M To identify half a shape. F To put together halves to make whole shapes. F To break an object in half. F Uses mathematical terms to describe 3d shapes. GS
	Key: Number and Place Value NPV , Addition A , Subtraction S , Multiplication Mx , Division D , Fractions F , Measurement M , Geometry Shape GS , Geometry Position GP											

MATHEMATICS YEARLY OVERVIEW
YEAR 1



Term	1	2	3	4	5	6	7
Autumn	Number and place value	Number and place value	Addition	Addition	Subtraction	Subtraction	
	Number and place value	Measure – Money	Addition and Subtraction <i>(context money)</i>	Measure – Length	Addition and Subtraction <i>(context length)</i>	Geometry – Properties of shapes	Statistics
Spring	Measure - Time	Number and place value	Addition/ Subtraction	Addition/ Subtraction	Measure – Capacity and Mass	Addition and Subtraction <i>(context capacity and mass)</i>	
	Geometry – Position and direction	Addition	Multiplication	Division	Fractions		
Summer	Measure – Time	Geometry – Properties of shapes	Number and place value	Measure – Money	Addition/ Subtraction	Addition/ Subtraction	
	Addition/ Subtraction	Addition/ Subtraction	Multiplication	Multiplication	Division	Fractions	Statistics

MATHEMATICS YEARLY OVERVIEW
YEAR 2



Term	1	2	3	4	5	6	7
Autumn	Number and place value	Number and place value	Addition/ Subtraction	Addition/ Subtraction	Addition/ Subtraction	Measure - Time	
	Geometry – Properties of shapes	Fractions	Division	Multiplication	Statistics	Measure – Money	Addition and Subtraction <i>(context money)</i>
Spring	Number and place value	Addition/ Subtraction	Addition/ Subtraction	Multiplication/ Division	Multiplication/ Division	Measure - Time	
	Measure – Length	Addition and Subtraction <i>(context length)</i>	Multiplication and Division <i>(context length)</i>	Fractions	Geometry – Position and direction		
Summer	Geometry – Properties of Shapes	Number and place value	Measure – Capacity and Mass	Addition and Subtraction <i>(context capacity and mass)</i>	Multiplication and Division <i>(context capacity and mass)</i>	Measure – Time	
	Measure – Length	Four Operations <i>(context measure)</i>	Four Operations <i>(context money)</i>	Fractions	Statistics	Geometry – Position and direction	Four Operations <i>(context measure)</i>

MATHEMATICS YEARLY OVERVIEW
YEAR 3



Term	1	2	3	4	5	6	7
Autumn	Number and place value	Addition and Subtraction	Addition and Subtraction	Multiplication and Division	Multiplication and Division	Measure – Time	
	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Geometry – Properties of shapes	Statistics	Measure – Volume and capacity	Measure – Length and mass	Four Operations (<i>context volume, capacity, length and mass</i>)
Spring	Number and place value	Geometry – Properties of shapes	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Addition and Subtraction	Multiplication and Division	
	Statistics	Measure – Money	Four Operations (<i>context money</i>)	Measure – Time	Four Operations		
Summer	Number and place value	Addition and Subtraction	Multiplication and Division	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	
	Measure – Volume and capacity	Four Operations (<i>context volume and capacity</i>)	Measure – Length and mass	Four Operations (<i>context length and mass</i>)	Geometry – Properties of shapes	Statistics	Measure – Time

MATHEMATICS YEARLY OVERVIEW
YEAR 4



Term	1	2	3	4	5	6	7
Autumn	Number and place value	Addition and Subtraction	Addition and Subtraction	Multiplication and Division	Multiplication and Division	Measurement – Time	
	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Geometry – Properties of shapes	Statistics	Measurement – Length and mass	Measurement – Volume and capacity	Four Operations (<i>context volume, capacity, length and mass</i>)
Spring	Number and place value	Addition and Subtraction	Multiplication and Division	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Geometry – Position and direction	
	Statistics	Measurement – Money	Four Operations (<i>context money</i>)	Measurement – Time	Geometry – Properties of shapes		
Summer	Number and place value	Addition and Subtraction	Multiplication and Division	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Geometry – Position and direction	
	Measurement – Volume and capacity	Four Operations (<i>context volume and capacity</i>)	Measure – Length and mass	Four Operations (<i>context length and mass</i>)	Geometry – Properties of shapes	Statistics	Measurement – Time

MATHEMATICS YEARLY OVERVIEW
YEAR 5



Term	1	2	3	4	5	6	7
Autumn	Number and place value	Addition and Subtraction	Multiplication and Division	Multiplication and Division	Four Operations	Measurement – Time	
	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Geometry – Properties of shapes	Measurement – Length and mass	Measurement – Volume and capacity	Four Operations (<i>context volume, capacity, length and mass</i>)	Statistics
Spring	Number and place value	Addition and Subtraction	Multiplication and Division	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	
	Measurement – Money	Four Operations (<i>context money</i>)	Measurement – Time	Geometry – Position and direction	Geometry – Properties of shapes		
Summer	Number and place value	Addition and Subtraction	Multiplication and Division	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	
	Statistics	Geometry – Position and direction	Geometry – Properties of shapes	Measurement – Volume and capacity	Measurement – Length and mass	Measurement – Money	Four Operations (<i>measurement</i>)

MATHEMATICS YEARLY OVERVIEW
YEAR 6



Term	1	2	3	4	5	6	7
Autumn	Number and place value	Addition and Subtraction	Multiplication and Division	Multiplication and Division	Four Operations	Measurement – Time	
	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Geometry – Properties of shapes	Measurement – Volume, capacity and mass	Measurement – Length and money	Four Operations (measurement)	Statistics
Spring	Fractions, Decimals and Percentages	Fractions, Decimals and Percentages	Four Operations	Four Operations	Algebra	Assessment Week – move accordingly	
	Four Operations (context money)	Measurement – Time	Geometry – Position and direction	Geometry – Properties of shapes	Ratio and Proportion		
Summer	Number and place value	Four Operations (money)	Algebra	Fractions, Decimals and Percentages	Geometry – Properties of shapes Ratio and Proportion	Statistics	
	Four Operations	Geometry – Position and direction	Geometry – Properties of shapes	Measurement – Volume, capacity and mass	Measurement – Length and money	Four Operations (measurement)	Assessment Week – move accordingly

National Curriculum Strands





Chapter 1

Number and Place Value



EYFS 1 – Number and place value (When planning ensure you track forwards to year 1)

Early Learning Goal 11

Children count reliably with numbers from 1 to 20 and place them in order.

Key Vocab: number, zero, one, two, three etc, none, how many?, count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, guess how many, estimate, nearly, close to, about the same as, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between.

Key concepts

When there are more objects the group gets bigger. When there are fewer (less) objects the group gets smaller.

When counting a group the last number represents the quantity.

Anything can be counted: claps, steps, jumps...

There are many numbers in the world around us.

When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.

Potential barriers/misconceptions

Pupils show confusion in vocabulary- more / less.

Misconception can occur through the linking of words- the bigger a number (in size) the greater it's quantity. i.e.

3 is bigger than **7**. (worth more than...)

Some pupils at this stage cannot differentiate between numbers and letters.

Pupils may be able to recite number words up to ten but do not count objects with 1 to 1 correspondence.

There may be little understanding of the value that each number holds

Pupils find it challenging to identify 'same' and 'different when working visually as they don't focus on the detail.

Example Questions

If we count around the circle starting with Gemma, who will say 5?

Look at the bowl of apples. Are there more green apples or red apples? How can you find out?

The birthday card has a 4 on it. Raza is four today. Put the right number of candles on his birthday cake.

How many counters are there?



Select the correct number card and match it with the counters.



Count with me to 10. One, two, three....

Count these buttons. You can move them as you count them if you wish.

What number is the one before six?

Put three coats up on the pegs

Bring me five aprons. Can you put one back?

Are there more books on the top shelf or on the bottom shelf? How do you know?

Which set has more cubes? The set of red cubes or the set of green cubes? How do you know?

Show a card. 'Read me the number on this card'.

Look at this telephone. Can you press the number 4? Number 6?

Learning objectives (see overleaf for exemplification)

To make comparisons between quantities.

To use language of quantities such as 'more' and 'a lot'.

To use the language of 'more' to compare sets of objects.

Recite number names in sequence 0-10.

Select a small number of objects from a group 'give me one, two etc'.

To compare two groups of objects (identifying 'the same').

To use number names and language.

To match numeral and quantity correctly.

To use one to one correspondence (touches each object and gives it a number).

To count objects in a line.

To know that numbers identify how many there are in a set (triad).

To create and experiment with symbols and marks representing ideas of numbers, then numerals.

To use more/most and less/least.

Mental maths(can revisited throughout day once concept has explicitly shared)

One, two, three four five. Once I caught a fish alive...

One potato, two potatoes, three potatoes, four...

Higgledy, piggledy, my fat hen...

This old man, he played one...

Recite sequence 1,2,3 up to 10

Count objects: tiny things in a matchbox, pieces of a jigsaw, letters in your name etc.

Count in 2s: pairs of socks, pairs of animals

Count in 1s (say aloud every other number)

Recognise recitation errors: (could use a puppet)

One, two, four, five (word omitted)

One, two, four, three, five (words in the wrong order)

One, two, three, three, four (repeating a word)

Start from a given number name and stop at another (start with three, hold it in your head, count to six)

Recite the number names in order to 5 then 10

To count backwards from 10

To count backwards to zero (none) from any number

Estimate (guess) how many marbles in the jar, coins in a purse etc



EYFS 1 – Number and place value			Progression (a combination of these models and images can be used for every objective)		
To make comparisons between quantities.	To use language of quantities such as 'more' and 'a lot'.	To use the language of more to compare sets of objects.			
Which plate would you like? Why? 	 	 Which bowl has more fish in?			
Recite number names in sequence 0-10.	Select a small number of objects from a group 'give me one, two etc'.	To compare two groups of objects (identifying 'the same').	To use number names and language.		
			zero – none six one seven two eight three nine four ten five more, less bigger, smaller		
To match numeral and quantity correctly	To use one to one correspondence (touch each object and give it a number)	To count objects in a line			
		 There are five cars in the group.			
To know that numbers identify how many there are in a set. (triad- three key elements)	To create and experiment with symbols and marks representing ideas of numbers then numeral.	To find more/ most and less/least			
It looks like: It sounds like: SIX You make it like this:	Make own marks or tallies resulting from practical activities. Pictorial representations of groups. When beginning to record numbers: Trace with a finger cut out numerals Sandpaper Rough fabric Tin foil Corrugated card Make numbers in sand, foam, using plasticine.	<u>Isaac's bears</u> <u>Hussein's bear</u> Who has the <u>most</u> bears? Who has the <u>least</u> ? <div style="float: right; border: 1px solid blue; border-radius: 50%; padding: 10px; width: fit-content; margin-top: 20px;">Isaac has more bears than Hussain.</div>			



EYFS 2 – Number and place value (When planning ensure you track forwards to year 1)	
<p><u>Early Learning Goal 11</u> Children count reliably with numbers from 1 to 20 and place them in order.</p>	<p><u>KS1 ready:</u> Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number. Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s Given a number, identify one more and one less Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least Read and write numbers from 1-20 in numerals and words</p>
<p>Key vocab: number, zero, one, two, three etc, none, how many?, count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, guess how many, estimate, nearly, close to, about the same as, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between.</p> <p>Key Concepts: When there are more objects the group gets bigger. When there are fewer objects the group gets smaller. When counting a group the last number represents the quantity. Anything can be counted: claps, steps, jumps... There are many numbers in the world around me. We can write number with words and squiggles. When we are talking about objects we say ‘more than’ and ‘fewer than’. When we are talking about numbers we say ‘greater than’ and ‘smaller than’.</p>	<p>Learning objectives (see overleaf for exemplification) To count reliably from 1-20 To use one to one correspondence (touch each object and give it a number) To count objects in a line- beginning to count beyond 10 To count actions or objects without physically touching them. To count objects in a group/ irregular arrangement. (using first same objects/ then different objects) To represent numbers using fingers, marks and pictures. To recognise numerals (0-5), (0-10) and (0-20). To order numbers from 0-20. To select the correct numeral to represent 1-5 then 1-10 objects. Write numbers to 20. To estimate how many objects can be seen and check by counting. To recognise numbers in a group without counting out (subitise). To make ten (feel the ten-ness of ten).</p>
<p>Potential barriers/misconceptions Pupils show confusion in vocabulary- more / less. Misconception can occur through the linking of words- the heavier object is the one that is ‘higher’ on the balance (When using balances to compare quantity). Pupils may be able to recite number words up to ten but do not count objects with 1 to 1 correspondence. There may be little understanding of the value that each number holds</p>	<p>Mental maths (can revisited throughout day once concept has explicitly shared) To count from 1-20 To count from non-zero starting point (up to 20). To recite the number names in order, continuing the count from a given number Recognise recitation errors: (could use puppet) Thirteen, fourteen, fiveteen (not changing the pattern) Eighteen, nineteen, tenteen (error by analogy) Thirty-nine, thirty-ten (error by analogy)</p>
<p>Example Questions One, two, buckle my shoe, three, four, knock at the door. Which two numbers come next? 10, 9, 8, 7, carry on counting backwards until ‘blast off!’ If we count round the circle starting at Lewis with 3, who will say 9? What number comes next after 12 when you count? Make a line of toy cars. Make the second car yellow and the fifth car red. Count on for me as far as you can go. One, two, three... What is the next number after four? I will say some numbers. I want you to count on the next three numbers. Four, five, six... Take this box of unifix. Count out nine of the unifix cubes and put them onto the table. Can you check? How many circles are there in the picture (show up to 5)? Tell me without counting them. Now check by counting. Guess how many cars there are on the table. (Place up to 10). Now check by counting them. Which plate has fewer biscuits on it? How do you know? There are 8 cubes in this stick of cubes. There are five cubes in this stick of cubes. Which stick has more cubes? Ella has five apples, Dom has three apples. Who has fewer apples? Ella or Dom? Choose two cards from this set. Which of your two numbers is more? Which number is less?</p>	<p>Start from a given number name and stop at another. (start with 2, hold it in your head, count on to 8) Count on several numbers from a given number (using fingers to help: count on three numbers from 4) To use ordinal numbers in different contexts (Who is third in the line?) To say the number name that goes before a given number name. (What number comes before □?) Recite the number sequence consistently back to zero from any given number to 20 To count in 2s, 5s and 10s. To count on in tens from any given tens number. (Count on in tens from 30) To count back in tens from any given tens number. Estimate the number in a group (how many children in class today?) To know which number is worth more/less. To say the number that is one more/one less than the given number. Say a number lying between two given numbers. Begin to use and understand ordinal numbers in different contexts (first, second, third..... & last)</p>



EYFS 2 – Number and place value Progression (a combination of these models and images can be used for every objective)

To count reliably from 1-20	To use one to one correspondence (touch each object and give it a number)		To count objects in a line (beyond 10)	To count actions or objects without physically touching them.
<p>one eleven two twelve three thir<u>teen</u> four four<u>teen</u> five fif<u>teen</u> six six<u>teen</u> seven seven<u>teen</u> eight eigh<u>teen</u> nine nin<u>teen</u> ten twen<u>ty</u></p>			<p>11 There are eleven cars in the group</p>	<p>Listening to the number of claps. The rings of a bell. Jumps in the air. Children at the front of the class. Windows on a building</p>
To count objects in a group/ irregular arrangement	To represent numbers using fingers, marks and pictures		To recognise numerals (0-5), (0-10) and (0-20)	
<p>There are nine cars in the group</p>		<p>'Put the silver car in space 8'</p>		
To select the correct numeral to represent 1-5 then 1-10 objects	To write numbers to 20	To estimate how many objects can be seen and check by counting	To recognise numbers in a group without counting out (subitise)	To make ten* (recognise the ten-ness of 10) (*see addition for number bonds within 10)
	<p>Trace. Write in the air. Paint. Model in play dough. Make number signs for classroom.</p>	<p>How many oranges do you think there are? Can you check?</p>	<p>Can recognise iconic, regular images of small numbers (e.g. dice patterns).</p>	<p>Egg box ten Unifix ten bundles of ten straws</p>



Year 1 – Number and place value (When planning ensure you track back to Reception and forwards to year 2)

<p><u>National Curriculum</u> Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number. Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s Given a number, identify one more and one less Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least Read and write numbers from 1-20 in numerals and words</p>	<p><u>Notes and guidance (non statutory)</u> Pupils practice counting (1,2,3...) ordering (e.g. first, second, third) and to indicate a quantity (e.g. 3 apples, 2 centimetres), including solving simple concrete problems until they are fluent. Pupils begin to recognize place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations. They practice counting as reciting numbers and counting as enumerating objects, and counting in 2s, 5s, and 10s from different multiples to develop their recognition of patterns in the number system (e.g. odd/even) including varied practice through increasingly complex questions. They recognize and create repeating patterns with objects and with shapes.</p>
<p>Key vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, regroup, fair swap, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between, half way. Key Concepts A group of objects can be partitioned in a number of ways- the total stays the same. (conservation) Ordinal numbers are for describing the position in a group of objects. When comparing we use the terms ‘greater than’ and ‘smaller than’ and ‘more than’ and ‘fewer than.’ When we are talking about objects we say ‘more than’ and ‘fewer than’. When we are talking about numbers we say ‘greater than’ and ‘smaller than’. Making ten first supports with number conservation. Children then count on from ten rather than starting at 1.</p>	<p>Learning objectives (see overleaf for exemplification) To identify one more and one less. To compare quantities (using equal to, more than, less than (fewer), most, least) To match numbers and quantities. To locate numbers on a number line. To read & write numbers from 1-20 in numerals and words. To identify odd and even numbers. To understand ordinal numbers. To compare numbers up to 20 (and beyond). To describe and extend number sequences. To make ten. To regroup (carry out a fair swap). To make ten and count on (in concrete). To identify ten and count on (in pictorial). To count out a 2 digit number to 20 and regroup in the 1s. To partition and recombine numbers to 20 into 10s and 1s (teen numbers). To partition and recombine any 2 digit number into 10s and 1s.</p>
<p>Potential barriers/misconceptions Unable to recognise numbers. Knowledge of saying numbers out loud with no concern for value or amount of objects. No understanding of the value that each digit holds. Understanding of number size – confusion over 3 is bigger than 1. Does not count with 1-1 correspondence. Able to count forwards but struggles to count backwards or find ‘one less than...’ Counts all rather than counting ‘on’ (no conservation of number). Sees a ‘ten’ as one rather than ten ones. Confusion between ‘teen’ numbers and multiples of ten: 16, 60. Reversal of digits.</p>	<p>Mental maths To count to and across 100 To count larger collections by grouping into tens, then fives or twos. To count backwards in ones from any two digit number To count on any given single digit number from any two digit number (count on seven from 22) To count in multiples of 2, 5 and 10 To count on in tens from a tens number stopping at a given number. (count from 20 to 60) To count back in tens from a tens number stopping at a given number (count from 80 back to 30) To describe and extend number sequences: counting on or back in steps of ones or tens from any given number. Count in 2s from 0-20, count in 2s from any given number To identify one more and one less than any given number Can say whether any number from 1-100 is odd or even and why. Count in tens from zero... from 40... from 8 Count in 2s from zero, count from 1,3,5 To say what number comes next in a given pattern. (16,14,12, □,□) To recall number bonds (see addition strand for exemplification) To know number bonds of all numbers within 10 (6 = 1+5, 5+1, 4+2, 2+4 etc) To know number bonds to 10, To know number bonds within 20 To make a reasonable estimate (then count to check) To state the value of the digits in a two digit number (14 is one ten and four ones)</p>
<p>Example Questions What number comes after 22? Before 65? Count back from 10 to six. How many did you count? Which tens number comes after 50? Before 80? What would be the best way to count marbles into the jar? There is always 1 left over when an odd number is divided by 2. True or false? Can you prove it? Draw a ring around the person who is 9th in the line. Estimate the number of pencils. Estimate how many pairs of socks you could make. (Show a picture of unpaired socks) Look at these numbers: 34 12 45 60 72 28 Which of these numbers is the largest? Which of these numbers is between 10 and 20? This sentence is correct: 8 is less than 10. Two of these sentences are correct. Tick them: 18 is more than 30, 26 is less than 60, 50 is more than 17, 47 is less than 21. Fill in the blanks: 35 is more than □, 35 is between □ and □, 35 has □ tens. Write the number thirty-two. Fill in the missing numbers: 18 is 1 less than □, 18 is 10 less than □.</p>	



Year 1 – Number and place value Progression (a combination of these models and images can be used for every objective)

<p>To find one more/less than a given number</p>	<p>To compare quantities</p>	<p>To match numbers and quantities.</p>	<p>To locate numbers on a number line.</p>	<p>To read and write numbers to 20</p>
<p> One more than five is six. </p>	<p>'More than' to compare objects. 'There are more green apples than red apples' 'There are fewer/less red apples than green apples' </p> <p>'Greater than' to compare numbers. 5 is greater than 3 3 is smaller than 5 Count and compare</p>	<p>9 Nine </p> <p>Can you make this amount using unifix cubes?</p>	<p>Where would 15 be on the number line? How do you know? _____ 10 _____ 20 _____ 30</p> <p>On a bead string? On a counting stick? </p>	
<p>To identify odd and even numbers</p> <p> 1 odd 2 even 3 odd 4 even 5 odd</p> <p>Represent up to 9 using ten grid.</p> <p> Use 'pairs of' to represent even</p> <p></p>	<p>To understand ordinal numbers</p> <p>Circle the fourth elephant </p> <p>1st</p> <p>Circle the ninth spider 1st</p>	<p>To compare numbers up to 20. (fewer/more) (smaller/greater)</p> <p>Which set has fewer? A B Set A has 5 spiders and set B has 12 spiders. Set A has 7 spiders fewer than set B.</p> <p>17 13 17 is greater than 13.</p>		
<p>To describe and extend number sequences</p> <p>How many stars in the next pattern? </p> <p>Find the missing numbers: 15, 14, 13, □, □, □</p>	<p>To make ten</p> <p>To use bundles of straws for children to feel the 'ten-ness' of ten.</p> <p> One ten and 2 ones = 12</p> <p></p>	<p>To regroup (carry out a fair swap)</p> <p>For children to use Dienes to create a 'fair swap' (regrouping of ten ones for one ten)</p> <p> Fair swap</p> <p></p>		
<p>To make ten and count on (concrete)</p>	<p>To make ten and count on (pictorial)</p>	<p>To count out a 2 digit number to 20 and regroup in the 1s</p>	<p>To partition and recombine numbers to 20 into 10s and 1s. (ten numbers then beyond 20)</p>	
<p>Use objects, dienes or bundles of straws to 'make 10' then count on:</p> <p></p> <p>10.....11, 12, 13, 14 11 10.....11, 12 14 10.....11, 12 12</p>	<p>Draw around ten and then count on.</p> <p> Ten and eight is eighteen.</p> <p>10..... 11,12,13,14,15,16,17,18</p>	<p></p>	<p>'16 is 1 ten and 6 ones'</p> <p></p>	



Year 2 – Number and place value (When planning ensure you track back to year 1 and forwards to year 3)

<p>National Curriculum Count in steps of 2,3, and 5 from 0 and in tens from any number, forward and backward Recognise the place value of each digit in a two-digit number (tens, ones) Identify, represent and estimate numbers using different representations including the number line. Compare and order numbers from 0 up to 100; use <, > and = signs Read and write numbers to at least 100 in numerals and in words Use place value and number facts to solve problems.</p>	<p>Notes and guidance (non statutory) Using materials and a range of representations, pupils practice counting, reading, writing and comparing numbers to at least 100 and solving a variety of related problems to develop fluency. They count in multiples of three to support their later understanding of a third. As they become more confident with numbers up to 100, pupils are introduced to larger numbers to develop further their recognition of patterns within the number system and represent them in different ways, including spatial representations. Pupils should partition numbers in different ways (for example, $23 = 20 + 3$ and $23 = 10 + 13$) to support subtraction. They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder.</p>
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<p>Key Vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, regroup, fair swap, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between, half way, place, place value, stands for, represents, round, nearest, estimate.</p> <p>Key Concepts Numbers can be partitioned in many ways into part, part, whole. (Unique partitioning is when numbers are broken up in the standard representation i.e. 63 is 6 tens and 3 ones. Multiple partitioning is the ability to also see: $63 = 5$ tens and 13 ones or 2 tens and 43 ones- this is an important tool for mental strategies) We can regroup ten ones for one ten. Ten tens is one hundred. In a two digit whole number the digit indicating the multiple of 10 is written on the left, and that to distinguish between, say 20 and 2, a zero is put in the space on the right as a place holder. Zero is a place holder and means 'no ones, no tens, no hundreds etc.' Numbers can be compared using the terms 'greater than' and 'smaller than' with and without concrete representation.</p>	<p>Learning objectives (see overleaf for exemplification) To represent 2 digit numbers (concrete) To count within 100 by making tens first. To recognise the place value of each digit in a 2 digit number. To compare numbers from 0 – 100. To order numbers from 0-100. To partition and recombine 2 digit numbers into 10s and 1s. To partition and recombine 3 digit numbers into 100s, 10s and 1s. To partition numbers in different ways. Identify numbers on a number line. To use the greater than, less than and equals signs (<,> , =) To begin to round numbers less than 100 to the nearest 10. Read and write numbers in numerals and words.</p>
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<p>Potential barriers/misconceptions Reversal of digits 03 for 30 and 31 for 13. This can create problems when ordering numbers. Confusion about the place value of numbers. Difficulties especially apparent when ordering numbers such as 212 and 221. Failure to understand that the position of the numeral gives it the value. Pupils not always sure what makes a 'sensible' answer (not estimating).</p>	
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<p>Example Questions Here are some numbers: 44 87 62 28 51. Write them in order; the first one is done for you: 28 □ □ □ □ . Here are two signs: '<' '>' '<'. Use the signs to make the following correct: $54 \square 16$, $19 \square 94$, $51 \square 35$ Ben puts 12 coins on a table. He hides some of them under his hand. How many coins is Ben hiding? Tim bought two pieces of fruit. He spent thirty pence altogether. He bought an orange for 12 pence. What did he pay for the other piece of fruit? Circle two numbers that add to make a multiple of 10: 11 12 13 14 15 16 17 18 19 Fill in the blank to make this correct: $40 - 30 = 10 + \square$. Write the two missing numbers in this sequence: $\square 45 47 49 51 \square 55 57$ Charlie is making 3-digit numbers with these cards. He can make this number: 7 2 4. Write all the other 3-digit numbers he can make. Write the missing digits to make this correct: $\square 0 + 3 \square = 43$ Write an odd number between 34 and 44. Write the missing numbers in this sequence: 47 42 37 □ □ 22 17 12 Write a number in the space to make this correct: $867 = \square + 60 + 7$ Sarah has 60 sweets. She puts 6 sweets in each party bag. How many bags does she put sweets in?</p>	<p>Mental maths To count in steps of 2, 3, 5 and 10 (forwards and backwards from any given number). To count on in tens from any given number (with and without a hundred square). To count to and across 100 from any given number (forwards and backwards). To have rapid recall of the x2, x3, x5 and x10 tables. To count up in threes from any given number (forwards and backwards). To find ten more than a multiple of ten (ten more than 40). To identify 1, 10, or 100 more/less than any given number. To accurately say the sequence of odd numbers from 1-19. To say whether any number is odd or even. To recognise multiples of 10, 5 and 2 and say how they know. To know the value of each digit (what is the number equivalent to 6 tens and 5 ones?). To know number bonds within 10 (for number bond exemplification see addition strand). To know number bonds to 10. To know number bonds within 20. To use the language of ordinality up to twentieth.</p>
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Year 2 – Number and place value Progression (a combination of these models and images can be used for every objective)

To represent 2 digit numbers (concrete)
(for regrouping and making 10 see year 1)

Count 'bundles' of 10: 5 tens = 50

Make 2 digit numbers with dienes: 4 tens + 4 ones = 44

To count within 100 by counting tens first.

10, 20, 30, 40, 50, 60..... 61, 62, 63, 64, 65, 66, 67, 68
There are 68 in the group.
60 and 8 make 68.

To represent numbers as tens and ones in a place value table.

$68 = \square \text{ tens} + \square \text{ ones}$
 $68 = 60 + 8$

Tens	Ones
6	8

What are the missing numbers? Can you make these numbers? 97, 62, 33, 48...

To compare numbers

Which number is greatest?
How much greater is it?

If the tens are equal then we compare the ones.

25 is 3 more than 22
22 is 3 less than 25

To order numbers from 0-100

Tens ones
2 3
2 5
3 3

Abstract: pupils to place range of numbers in order.

To partition and recombine 2 digit numbers into 10s and 1s. (unique partitioning)

$57 = 50 + 7$

To partition and recombine 3 digit numbers into 100s, 10s and 1s. (unique partitioning)

$173 = 100 + 70 + 3$

To partition numbers in different ways. (multiple partitioning)

64 can be partitioned as = 6 tens and 4 ones (60+4)
64 can also be partitioned as:

Identify numbers on a number line.

Locate numbers on a number line

What number is marked by the arrow? How do you know?
Can you locate: 22, 39, 94.....on the number line?

On an unmarked number line?

To use the greater than>, less than< and equals= signs (<,>, =)

$22 < 54$

To begin to round numbers less than 100 to the nearest 10.

78 to nearest 10

Look at the ones
5 or above? – round on to the next tens number
Less than 5? – round back to the previous tens number

Read and write numbers in numerals and words. (to 100 and beyond)

one	twelve
two	thirteen
three	fourteen
four	fifteen
five	sixteen
six	seventeen
seven	eighteen
eight	nineteen
nine	twenty
ten	twenty-one
eleven	twenty-two

Note that when recording numbers over 20:
22
twenty-two
(this is written with a hyphen between the tens & ones)

Key steps in developing understanding of place value

- 1) Ten- 1 ten has a different value to 1 one
- 2) Tens and ones- Tens and ones can exist side by side
- 3) Number names- Instead of naming a number 1 ten and 4 ones we give it a name-fourteen
- 4) Hundreds, tens and ones- Children should be aware that when they have ten tens they must 'regroup' to make 1 hundred
- 5) Numbers can be broken into different parts. The number 36 can be 3 tens and 6 ones. It is also 2 tens and 16 ones



Year 3 – Number and place value Progression (a combination of these models and images can be used for every objective)

To represent 3 digit numbers (concrete) **To find 1, 10 or 100 more than a given number (concrete).**

10 tens = 1 hundred

10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
(ten hundreds = 1 thousand)

one hundred and three = 103

two hundred and twenty-one = 221

Count on one

Hundreds	Tens	Ones
4	1	5

$221 + 1 =$ one more than 221 is 222

$221, 222$

Count on ten

Hundreds	Tens	Ones
2	2	1

$221 + 10 =$ ten more than 221 is 231

$221, 231$

Count on one hundred

Hundreds	Tens	Ones
2	2	1

$221 + 100 =$ one hundred more than 221 is 321

$221, 321$

To recognise the place value of each digit in a three digit number.

Hundreds	Tens	Ones
2	1	5

Stands for : 2 hundreds or 200 1 ten or 10 5 ones or 5

To use part, part whole to partition numbers in different ways.

$146 = 100 + 40 + 6$ or $146 = 130 + 16$

To compare numbers up to 1000

338

334

First compare the hundreds, then the tens, then the ones
338 is greater than 334 (334 is smaller than 338)

To order numbers up to 1000

Arrange these numbers in order. Begin with the smallest. 476, 259, 601

Hundreds	Tens	Ones
4	7	6
2	5	9
6	0	1

601 is greater than 476 and 259.
476 is greater than 259.
In order from smallest: 259, 476, 601

Identify, represent and estimate numbers up to 1000 in numerals and words.

500 ↓ 1000

What number is here on the number line? How do you know?

To recognise the place value of different measures.

Write numbers in words:
999 = nine hundred and ninety-nine
234 = two hundred and thirty-four
303 = three hundred and three

139cm = 100 cm + 30 cm + 9 cm (100cm = 1 metre)

To use Dienes and coins to understand place value

The idea of 'regrouping' can be reinforced through the exchanging of coins.

In 506 = 5 hundreds, 0 tens and 6 ones
 $506 = 500 + 0 + 6$
In 506: The digit 6 is in the ones place, the digit 0 is in the tens place and the digit 5 is in the hundreds place.

214 =

Hundreds Tens ones

Pupils indicate: the value of each digit and the place at which it is situated.



Year 4 – Number and place value (When planning ensure you track back to year 3 and forwards to year 5)

<p><u>National curriculum</u> Count in multiples of 6,7,9,25 and 1000 Find 1000 more or less than a given number Count backwards through zero to include negative numbers Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens and ones) Order and compare numbers beyond 1000 Identify, represent and estimate numbers using different representations Round any number to the nearest 10, 100, 1000 Solve number and practical problems that involve all of the above and with increasingly large positive numbers. Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.</p>	<p><u>Notes and guidance (non-statutory)</u> Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice. They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far. They connect estimation and rounding numbers to the use of measuring instruments. Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.</p>
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Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property.
Key concepts
 In a two digit whole number the digit indicating the multiple of 10 is written on the left, and that to distinguish between, say 20 and 2, a zero is put in the space on the right as a place holder.
 Zero is a place holder and means ‘no ones, no tens, no hundreds etc.’
 Negative numbers can be found on a temperature scale.
 Negative numbers are below zero and the size of the negative number indicates the distance it is from 0.
 < is less than and > is greater than. When we multiply by 10/100 the place value of the digits change.
 Rounding numbers can alter a situation (i.e. results of a race etc.) A number line can be used to visualise decimals.

Potential barriers/misconceptions
 Ordering numbers is challenging if pupils don’t have a strong understanding of place value.
 Trouble giving values to each of the digits. In 23 the value of the first number is not 2 it is 2 tens or 20.
 Confusion around zero as a place holder.
 Pupils move into abstract too quickly and although they can recognise and read numbers up to 1000 they are unfamiliar with the place value of each digit.
 When multiplying by 10/100 children think they just ‘add zero’ without understanding that the place value of the digits have changed and zero then becomes a place holder.

Example Questions
 The sum of two numbers is 100. Write the missing digits: $3 \square + \square 3 = 100$
 Each missing digit in this sum is a 9 or a 1. Write in the missing digits. $\square\square + \square\square = 201$
 Paul says, “Every multiple of 5 ends in 5”. Is he correct? Explain how you know.
 Write in figures the number five thousand and thirty-two
 Jet has these numbers: 1330 1303 1033 1003 1030. She writes them in order from smallest to largest. What is the fourth number she writes?
 The temperature in London is 3°C. Paris is 9 degrees colder than London. What is the temperature in Paris?
 Circle the numbers nearest to 1000. 1050 1340 1046 1004 1040
 Match 3500 to numbers with the same value: 35 hundreds 3500 ones 35 tens 350 tens 350 hundreds.
 Write these prices in order from smallest to largest: 97p £11.50 £0.76 £8 £3.05
 Write these amounts in order in the boxes: £60.06 £60.60 £6.60 £6.06
 John makes a sequence of numbers. His rule is: “find half the last number, and then add 10”. Write the next two numbers in his sequence: 36 28 24 __ __
 Circle the number that is about the same as the correct answer to $49 + 48$. 10 50 40 100 70 200

Learning objectives (see overleaf for exemplification).
 To represent 4 digit numbers (concrete- place value counters).
 To find 1, 10, 100 or 1000 more than a given number (concrete).
 To recognise the place value of each digit in a four digit number.
 Order numbers beyond 1000.
 Compare numbers beyond 1000.
 Round any number to the nearest 10, 100, 1000. (To round appropriately given context see division strand)
 To identify and count in negative numbers.
 To estimate and round numbers using measuring instruments.
 To understand the history of different numeration systems.
 To read and understand Roman numerals.
 To understand the place value of decimals and fractions (see learning objectives in these strands).

Mental maths
 To count in multiples of 6,7 and 9
 To count in multiples of 25 and 1000
 To count backwards through zero to negative numbers.
 To find 1,10, 100, 1000 more than any given number (with 4 or more digits)
 To find 1,10, 100, 1000 less than any given number (with 4 or more digits)
 To multiply by 10, 100 and 1000 (understanding that digits move to the left when multiplied by 10...)
 To know what the value of each digit is up to 10,000.
 To count on from any given number crossing boundaries (count on 7 in ones from 669, 70 in tens from 669, 700 in hundreds from 669, 7000 in thousands from 2669).
 To round any two or three digit number to the nearest 10 or 100.
 To round measurements in seconds, minutes, hours, metres, kilometres, litres to the nearest 10 or 100 units.
 Estimate calculations by approximating. $(608+297 = 610+300 = \text{approximately } 910)$
 Approximate multiplications $(19 \times 16 = 20 \times 16 = (2 \times 16) \times 10 = 320)$
 Extend and explain number sequences (48, 41, 34, 27...) continuing beyond zero.
 To notice a pattern when counting from zero in 2s, 4s then 8s (4s are double 2s, 8s are double 4s)
 To recognise odd and even numbers up to 10,000 and make general statements about them. (if you add odd numbers the answer is even. Check. Explain why?)



Year 4 – Number and place value

Progression (a combination of these models and images can be used for every objective)

To represent 4 digit numbers (concrete- place value counters)

Using place value counters:
 = 2341
 (Pupils to regroup tens 1s for 1 ten counter etc.)

To find 1, 10, 100 or 1000 more than a given number (concrete).

Count on by tens

$2341 + 10 =$ ten more than 2341 is 2351
 $+ 10$
 2341, 2351

Count on by hundreds

$2341 + 100 =$ one hundred more than 2341 is 2441
 $+ 100$
 2341, 2441

Count on by thousands

$2341 + 1000 =$ one thousand more than 2341 is 3341
 $+ 1000$
 2341, 3341

To recognise the place value of each digit in a four digit number.

Thousands	Hundreds	Tens	Ones
1000	200	10	5

Stands for : 1 thousand 2 hundred 1 ten 5 ones
 or 1000 or 200 or 10 or 5
 $= 1000+200+10+5 = 1215$

Compare numbers beyond 1000

Thousands	Hundreds	Tens	Ones
2	3	4	1
2	4	2	1

Which is greater/smaller?
 $= 2214$
 $= 2421$
 If the thousands are same compare hundreds.

Order numbers beyond 1000

Arrange these numbers in order. Begin with the smallest. 6476, 4259, 4601

Thousands	Hundreds	Tens	Ones
6	4	7	6
4	2	5	9
4	6	0	1

First compare the thousands
 6476 is greater than 4259 & 4601.
 4601 is greater than 4259.
 In order from smallest: **4259, 4601, 6476.**

Round any number to the nearest 10, 100, 1000

Round off 2157 to the nearest 10:

2157 is between 2150 and 2160. It is nearer to 2160 than to 2150. 2157 is 2160 when rounded off to the nearest ten ≈ 2160

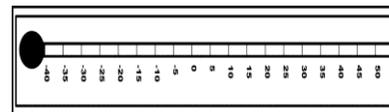
In 7506 = 7 thousands, 5 hundreds, 0 tens and 6 ones
 $7506 = 7000+ 500 + 0 + 6$
 In 7506: The digit 6 is in the ones place, the digit 0 is in the tens place, the digit 5 is in the hundreds place and the digit 7 is in the thousands place.

To identify and count in negative numbers.

Negative numbers are numbers that are less than zero.

Negative numbers < zero < positive numbers

Ensure thermometer is shown in both directions



To estimate and round numbers using measuring instruments

How much water is there? To the nearest 10ml
 What time is it to the nearest 5 minutes?
 How much does this weigh? To the nearest 100g?
 How long is this? To the nearest cm?



To understand the history of different numeration systems (Derek Haylock, Mathematics explained for primary teachers, 2006)

Egyptian Hieroglyphics	Roman numerals	Hindu-Arabic
I	I	1
IIII	V	5
X	X	10
CCCC	L	50
9	C	100
99 99 9	D	500
999 CCCC IIIII	CCCLXVI	366

Egyptian Hieroglyphic system (3000BC) had separate symbols for ten, hundred, thousand, ten thousand, a hundred thousand and a million. The Romans (3000 years later) based their numeration system on a similar system including additional symbols for 5, 50 and 500. The Hindu Arabic system (used today) uses fewer symbols and is based on the place value system. We know the value of the digit 2 based on where it is written i.e. 200, 20 or 2. The Roman system would record this as CC, XX or II. In our Hindu-Arabic place value system, all numbers can be represented using a finite set of digits: 0,1,2,3,4,5,6,7,8,9. The system uses ten as a base (ten fingers to count).

To read and understand Roman numerals.

1	4	7	10
ONE	FOUR	SEVEN	TEN
I	III	VII	X
2	5	8	11
TWO	FIVE	EIGHT	ELEVEN
II	V	VIII	XI
3	6	9	12
THREE	SIX	NINE	TWELVE
III	VI	IX	XII



L = 50
 C = 100
 D = 500
 M = 1000

MMXV = 2015



Year 5 – Number and place value (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum
 Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit.
 Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000.
 Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers including through zero.
 Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
 Solve number problems and practical problems that involve all of the above.
 Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

Notes and guidance (non-statutory)
 Pupils identify the place value in large whole numbers
 They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions they have met so far.
 They should recognise and describe linear number sequences (e.g. 3, 3, 4, 4 ...), including those involving fractions and decimals, and find the term-to-term rule in words (for example, add)

Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property, divisibility.
Key concepts
 If we need to work in the concrete to consolidate our understanding we can use place value counters.
 We work from left to right when determining the place value.
 We increase the powers of ten as we move from right to left.
 10 thousands = 1 ten thousands
 When counting in steps of powers of 10, we are multiplying by ten which changes the place value.
 The context for rounding is the most important element. Are pupils rounding up when buying packets of tiles for the floor (so as not to be short of tiles) or are they rounding back to the nearest 5 minutes when catching a train (so as not to miss it).
 The number line and use of ordinal numbers are useful when introducing the concept of negative numbers.
 To associate positive and negative integers the number line can be shown both horizontally and vertically.

Learning objectives (see overleaf for exemplification)
 To represent 6 digit numbers (to 1 000 000) (concrete- place value counters).
 To recognise the place value of each digit in a six digit number.
 To compare & order numbers to at least 1 000 000
 To recognise and describe linear number sequences.
 To find the term-to-term rule
 To interpret negative numbers
 To round numbers to the nearest 10, 100, 1000, 10 000 and 100 000 (To round appropriately in context see division strand)
 To count in steps of powers of 10 up to 1 000 000
 Read Roman numerals (See progression year 4)
 To understand decimals and fractions (see strands on decimals and fractions).

Potential barriers/misconceptions
 As the numbers increase, pupils find it difficult to read numbers aloud.
 When counting in 1000s pupils sometimes unsure what comes after 9000 (10 000). Also what comes before and after this number (9999, 10 001)
 Pupils do not make the link between these numbers and real life contexts. (Could be linked to populations of countries, costs of items etc.).
 Pupils counting in powers of ten are often 'ill advisedly' told that they need to 'add a naught'. Children must observe the transformation of numbers as the digits move into new place on the place value grid.
 -5 can also be referred to as 'negative 5' as 'minus 5' can suggest the need for subtraction.

Mental maths
 To count in steps of powers of 10 up to 1 000 000: 10, 100, 1000 etc (see exemplification over page).
 Count forwards and backwards with positive and negative whole numbers including through zero.
 Know the value of every digit in six digit+ numbers.
 To compare two numbers (which is less 4 thousands or 41 hundreds).
 To make the biggest/ smallest integer possible with a range of digits (i.e. 8 3 0 7 6 0 2).
 To know 1000, 10,000, 100,000 more/less than any six digit number.
 To multiply any number by 10, 100 and 1000 (and explain how the place value changes).
 To divide any number by 10, 100 and 1000 (and explain how the place value changes).
 To multiply decimal numbers by 10, 100 and 1000 (and explain how the place value changes).
 To divide decimal numbers by 10, 100 and 1000 (and explain how the place value changes).
 To identify the number that sits halfway between two numbers. (i.e. 27,400 and 28,000)
 To place six digit numbers in ascending and descending order.
 To look at a quantity (i.e. coins in a jar, grapes in a bowl) and make a reasonable estimate.
 To round any two, three, four digit number to the nearest 10, 100, 1000.
 To round measures. (i.e. distance between cities to the nearest km)
 To identify the best approximation.
 To calculate the rise and fall in temperature using both positive and negative integers.

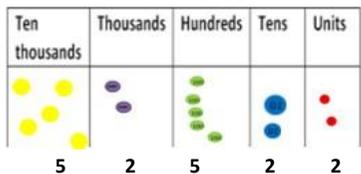
Example Questions
 □ and Δ each stand for a different number. What is their value? □ = 34 □ + □ = Δ + Δ + □
 Here are four digit cards. '7' '5' '2' '1' Choose two cards each time to make the following two-digit numbers. The first one is done for you: An even number - 52. A multiple of 9 - __ a square number - __ a factor of 96 - __
 In the number 15083, what does the 5 represent? Thousands, hundreds, tens, ones.
 A car costs more than £8400 but less than £9200. Tick the prices than the car could cost: £8397 £9190 £9230 £8999
 Round each number to the nearest whole number: 5.01 8.51 6.65
 James has 84 stamps. Emily has 57 stamps. Which of these is the BEST way to estimate how many stamps there are altogether: 90 + 60 = 150 80 + 60 = 140 80 + 50 = 130
 What does the digit 3 in 305 642 represent?

(This section contains the continuation of the 'Mental maths' list from the previous block, which has already been transcribed above.)



Year 5 – Number and place value Progression (a combination of these models and images can be used for every objective)

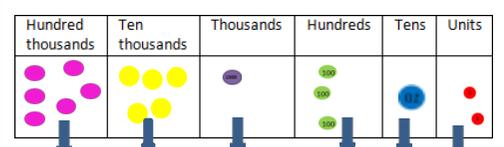
To represent 5 & 6 digit numbers (to 1 000 000) (concrete- place value counters).



Fifty-two thousand, five hundred and twenty-two

Children should be able to read numbers when given as digits and write numbers when given as words.

To recognise the place value of each digit in a six digit number.



In 651,312= six hundred and fifty-one thousand, three hundred and twelve.
 $651,312 = 600,000 + 50,000 + 1000 + 300 + 10 + 2$

To compare & order numbers to at least 1 000 000

When comparing numbers look at the value of each of the digits starting from the left. Arrange these numbers in order. Begin with the smallest. 16476, 14259, 14601

Ten Thousands	Thousands	Hundreds	Tens	Ones
1	6	4	7	6
1	4	2	5	9
1	4	6	0	1

First compare the Ten thousands
 16476 is greater than 14259 & 14601.
 14601 is greater than 14259.
In order from smallest: 14259, 14601, 16476.

To round numbers to the nearest 10, 100, 1000, 10 000 and 100 000

We round off numbers so that we are able to estimate. We use the approximation sign \approx to stand for **approximately equal to**. It shows rounding off of the numbers.

Round off to the nearest 100

What is 9872 rounded off to the nearest hundred?
 9872 is between **9800** and **9900**
 9872 is nearer to **9900** than **9800**
 9872 is **9900** when rounded off to the nearest hundred.
 $9872 \approx 9900$

To recognise and describe linear number sequences.

Use subtraction to identify if sequence is: **increase** or **decrease**

755,482 **705,482** **655,482** **605,482**

705,482 is **50,000** less than 755,482
 655,482 is **50,000** less than 705,482
 605,482 is **50,000** less than 655,482

50,000 less than 605,482 is 550,482
 The next number is **550,482**

To find the term-to-term rule

'To explain rule for a sequence and work out what comes next.'

Term	Term-to-term rule	Sequence
a) 1 st term = 3	Add 7	3, 10, 17, 24, 31
b) 3 rd term = 16	Subtract 9	-2, 7, 16, 25, 34
c) 5 th term = 3	Divide by 2	48, 24, 12, 6, 3
d) 1 st term = 5	Multiply by 2, add 1	5, 11, 23, 47, 95
e) 2 nd term = 2	Multiply by 4, divide by 2	1, 2, 4, 8, 16
f) 4 th term = 202	Multiply by 3, add 1	7, 22, 67, 202, 607

To interpret negative numbers

Familiar contexts such as temperatures, multi-story buildings, heights above and below sea level and bank balances can give meaning to positive and negative integers.

Round off to the nearest 100

What is 8276 rounded off to the nearest thousand?
 8276 is between **8000** and **9000**
 8276 is nearer to **8000** than **9000**
 8276 is **8000** when rounded off to the nearest thousand.
 $8276 \approx 8000$

To count in steps of powers of 10 up to 1 000 000

Larger whole numbers than 9 are constructed using powers of the base: ten, a hundred, a thousand etc.

Name of power	Numerals	Constructed from tens	Expressed as powers of ten in symbols	Expressed as powers of ten in words
A million	1000000	$10 \times 10 \times 10 \times 10 \times 10 \times 10 =$	10^6	Ten to the power six
A hundred thousand	100000	$10 \times 10 \times 10 \times 10 \times 10 =$	10^5	Ten to the power five
Ten thousand	10000	$10 \times 10 \times 10 \times 10 =$	10^4	Ten to the power four
A thousand	1000	$10 \times 10 \times 10 =$	10^3	Ten to the power three
A hundred	100	$10 \times 10 =$	10^2	Ten to the power two
Ten	10	$10 =$	10^1	Ten to the power one



Year 6 – Number and place value (When planning ensure you track back to year 5 for progression)

<p>National Curriculum Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit. Round any whole number to a required degree of accuracy Use negative numbers in context, and calculate intervals across zero. Solve number and practical problems that involve all of the above.</p>	<p>Notes and guidance (non-statutory) Pupils use the whole number system, including saying, reading and writing numbers accurately.</p>
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<p>Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property, factor, factorise, square, prime</p> <p>Key concepts The decimal point is used as the separator in the contexts of money and measurement. When counting in steps of powers of 10, we are multiplying by ten which changes the place value. When rounding numbers we talk about ‘rounding on’ and ‘rounding back’ to link to the number line/counting stick rather than rounding up and down. The context for rounding is the most important element. Are pupils rounding up when buying packets of tiles for the floor (so as not to be short of tiles) or are they rounding back to the nearest 5 minutes when catching a train (so as not to miss it)?</p>	<p>Learning objectives (see overleaf for exemplification)</p> <ul style="list-style-type: none"> To consolidate learning objectives from year 5 Then: To understand the place value of 7 digit numbers To identify negative integers. To calculate intervals across zero. To find the term-to-term rule To order and compare numbers up to 10 000 000 To round any whole number (To round appropriately given context see division strand) To extend place value to decimals To identify decimal numbers on a number line
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<p>Potential barriers/misconceptions Pupils counting in powers of ten are often ‘ill advisedly’ told that they need to ‘add a naught’. Children must observe the transformation of numbers as the digits move into new place on the place value grid. Some pupils may still write three hundred and forty seven as 30047, showing a lack of understanding around zero acting as a place holder. When looking at the number 300, the position of the three is what makes is 300, rather than the zeros. The function of the zero is to make this position clear and to signify no tens and no ones. Pupils may hear ‘tens’ and ‘hundreds’ if the wording is not articulated clearly when saying ‘tenths’ and ‘hundredths’.</p>	
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<p>Example Questions Imagine you have 25 beads. You have to make a three-digit number on an abacus. You must use all 25 beads for each number you make. How many different three-digit numbers can you make? Write them in order. Here are some digit cards: ‘2’ ‘4’ ‘6’ ‘6’ Write all the three digit numbers, greater than 500, that can be made using these cards. Tariq makes a sequence of 5 numbers. The first number is 2. The last number is 18. His rule is to add the same amount each time. Write the missing numbers: 2 ___ 18 Sarah is working with whole numbers. She says: “If you add 2 two-digit numbers you cannot get a four-digit number”. Is she correct? Explain why. The temperatures were: Inside: -2°C Outside: - 10°C What is the difference between these two temperatures? The temperature inside an aeroplane is 20°C. The temperature outside is - 30°C. What is the difference between these temperatures? Round each decimal to the nearest whole number: 5.01 8.51 7.75 Write half a million in figures. Write 2 and a half million in figures. Which two of these numbers when multiplied together have the answer closest to 70? 7.4 8.1 9.4 10 Write a decimal which is greater than 0.7 and less than 0.71 Write these numbers in order of size. Starting with the smallest: 1.01 1.001 1.101 0.11 Write down a multiple of 4 that is greater than one thousand.</p>	<p>Mental maths To count in multiples of any number up to x12 forwards and backwards from any given number. To count in steps of powers of 10 up to 1 000 000 (see exemplification year 5) To count in 11s, 15s, 19s, 21s, 25s then back. Can you go past zero? To count in steps of 0.1, 0.5, 0.25 to 10 then back. To multiply and divide whole numbers by 10, 100, 1000 To multiply and divide decimal numbers by 10, 100 and 1000 Count forwards and backwards with positive and negative whole numbers including through zero. Know the value of every digit in six digit+ numbers. To compare two numbers (which is less 4 thousands or 41 hundreds?). To make the biggest/ smallest integer possible with a range of digits (i.e. 8 3 0 7 6 0 2). To know 1000, 10,000, 100,000 more/less than any six digit number. To round any whole number to the nearest multiple of 10, 100 or 1000 To put integers in order from smallest to largest crossing zero. (-37, 4, 29, -4, -28) To make statements about identification of odd and even numbers. To find all the prime factors of any number to 1000 (the prime factors of 60 are 2,2,3 and 5, since 60= 2x30 = 2x2x15 = 2x2x3x5.) Use factors for finding products mentally (32x24 = 32 x 3 x 8 = 96 x 8 = 800 – (4 x 8) = 768 Identify numbers with an odd number of factors (squares) Identify two digit numbers with only two factors (primes) Recognise prime numbers.</p>
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Year 6 – Number and place value

Progression (a combination of these models and images can be used for every objective)

To understand the place value of 7 digit numbers.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1	6	4	9	0	0	0

One million, six hundred and forty-nine thousand.

In 1,649,000

The digit 1 stands for 1,000,000. The value of the digit 1 is one million
 The digit 6 stands for 600,000. The value of the digit 6 is six hundred thousand
 The digit 4 is in the ten thousands place. Forty thousand. (40,000)
 The digit 9 is in the thousands place. Nine thousand (9,000)

To identify negative integers.

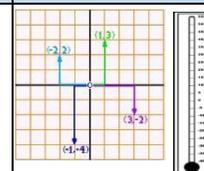
Concrete apparatus cannot be used to teach negative integers.



Positive and negative integers are seen as reflections in zero.

+3 to be referred to as 'positive three'.
 -3 to be referred to as 'negative three'.

To calculate intervals across zero.



Visual representations can be used to calculate the difference between positive and negative integers.



To find the term-to-term rule

Term	Term-to-term rule	Sequence
a) 1 st term = 3	Add 7	3, 10, 17, 24, 31
b) 3 rd term = 16	Subtract 9	-2, 7, 16, 25, 34
c) 5 th term = 3	Divide by 2	48, 24, 12, 6, 3
d) 1 st term = 5	Multiply by 2, add 1	5, 11, 23, 47, 95
e) 2 nd term = 2	Multiply by 4, divide by 2	1, 2, 4, 8, 16
f) 4 th term = 202	Multiply by 3, add 1	7, 22, 67, 202, 607

To order and compare numbers up to 10 000 000

When we compare numbers, we look at the value of each digit starting from the left.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1	7	1	2	9	3	5
1	7	1	2	8	4	6

9 hundreds is greater than 8 hundreds.
 So 1,712,935 is greater than 1,712,846

To round any whole number

We round off numbers so that we are able to **estimate**.

We use the approximation sign \approx to stand for **approximately equal to**. It shows rounding off of numbers.

Estimate the value of 6327×7 (round off the 4 digit number to the nearest thousand first)
 $6327 \times 7 \approx 6000 \times 7$
 $= 42000$

Estimate the value of $6742 \div 8$ ($6742 \div 8$ using knowledge of x8 tables)
 $6400 \div 8 = 800$
 $7200 \div 8 = 900$

6742 is nearer to 6400 than to 7200. So $6742 \div 8 \approx 6400 \div 8 = 800$.

To extend place value to decimals

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1000	100	10	1	0.1	0.01	0.001
$10 \times 10 \times 10$	10×10	10	1	—	—	—
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

To identify decimal numbers on a number line

1.35 can be explained in the context of length as 1 metre + 3 tenths of a metre + 5 centimetres

On the number line it lies between 1 and 2
 It lies between 1.3 and 1.4
 It lies between 1.34 and 1.36.

